INVESTIGATING CHILDREN'S ABILITY TO SOLVE MEASUREMENT ESTIMATION PROBLEMS

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In this study, how fourth- to sixth-grade children perform measurement estimation was investigated. The data were collected by a measurement estimation task that contained linear and area estimations and interviews from 72 children, each in one fourth- (n = 21), fifth- (n = 32), or sixth-grade (n = 19) class at local public elementary schools in cities in north Taiwan. The results indicated significant differences in the performance of the children in the estimation task among the three grade levels. The sixth-graders were observed to outperform the fourth-graders but they performed similarly to the fifth-graders. Grade level also influenced children's ability to estimate area rather than linear estimation. The children's strategies used for estimating an object with a long length included Benchmark, Guessing, Looking, and Other strategies.

INTRODUCTION

Measurement estimation, like problem solving, requires knowledge of measurement and the ability to use effective strategies to make reasonable estimates (Joram, Subrahmanyam, & Gelman, 1998). Children learn to perform measurement estimation by using visual perception at an early age (Sarama & Clements, 2009). However, elementary school children have been observed to be unsuccessful in making reasonable estimates (Chan, 2001; Forrester, Latham, & Shire, 1990).

In recent years, mathematics curricula and instruction in Taiwan (Taiwan Ministry of Education [TME], 2010) and other countries (e.g., National Council of Teachers of Mathematics [NCTM], 2000, 2006) have focused on measurement estimation. Although the importance of measurement estimation in mathematics education has attracted the attention of mathematics educators and researchers (Sarama & Clements, 2009), research on children's ability to perform measurement estimation remains inadequate.

Children's ability to perform measurement estimation is influenced by several factors such as the type of measure being estimated (e.g., linear or area), grade level (or age) (Forrester et al., 1990; Joram et al., 1998; Siegel, Goldsmith, & Madson, 1982), and problem contexts that require estimations (type of unit and quantity; Forrester et al., 1990). Generally, estimating length (or distance) is easier than estimating area (Chan, 2001). Children in higher grades were observed to be more successful in measurement estimation than those in lower grades (Siegel et al., 1982). However, some studies (Swan & Jones, 1980; Montague & Van Garderen, 2003) indicated that an increase in estimation ability was not necessarily positively associated with an increase in grade level.

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In addition to the aforementioned factors, Montague and Van Garderen (2003) suggested that curriculum and pedagogy may affect the ability of students to estimate in early grades. To strengthen students' mathematical power effectively, instructors must have a clear understanding of children's mathematical knowledge and skills. Thus, studies on the ability of children to estimate in varying grade levels are needed.

The present study extends previous studies on mathematical problem solving by exploring the estimation ability and strategies used by fourth- to sixth-grade children who received mathematics instruction that emphasized measurement estimations. This study focused on the ability of children to solve problems involving the estimation of linear and area measurements and addressed the following two research questions:

1. What are differences in the ability of children to perform linear estimations and area estimations among grade levels?
2. What are the strategies that children have adopted for estimating an object with a long length?

THEORETICAL FRAMEWORK

Mathematical Thinking Involved in Measurement Estimation

"Estimating" is the process in which a reasonable quantity or size of an object is provided without using measurement tools or measuring the object. The ability to perform measurement estimation involves multiple components, including estimating, approximating, and measuring, which lay the foundation for understanding physical measurement (Joram et al., 1998).

Furthermore, Carter (1986) and Joram, Subrahmanyam, and Gelman (1998) have purposed that developing a mental frame of reference for the sizes of units of measure requires constructing a mental structure that involves multiple cognitive processes of decomposing and re-compositing an object (or a numerical computation) to be estimated as well as comparing benchmark mental representations (e.g., physical references). Such measurement thinking is constructed based on sufficient knowledge of physical measurement and experiences in real measurement.

The Relationships Among the Types of Attributes To Be Estimated, Grade Level, and Ability To Perform Measurement Estimations

Joram et al. (1998) suggested that the basic unit-covering principle applies to both length and area measurement. Estimating linear measurement involves applying the unit-covering principle and mentally repeating units to estimate an object in one-dimension, such as length and distance. Children are able to compare length visually, which is the basis of length estimation, at a young age (Sarama & Clements, 2009). Moreover, knowledge of linear measurement and strategies for estimating lengths (e.g., guessing-and-checking) are frequently provided in the measurement curricula followed in early school years (NCTM, 2000, 2006; TME, 2010).

Area measurement, which involves two-dimensional spatial knowledge, requires knowledge of length measurement. Although not all estimation skills are developed similarly, regarding estimating area, applying the unit-covering principle and mentally repeating square units to estimate an object in two-dimensions is effective (Joram et al., 1998). For example, an area can be estimated by comparing the area to be estimated directly to one of the standard units of area. However, because the complexity of measuring length and area is different, studies have determined that children perform more successfully in linear estimation than in area estimation (Chan, 2001).

Forrester, Latham, and Shire (1990) determined that greater familiarity with measurement procedures and strategies related to numerical calculations improves children's competence in measurement estimation. Moreover, Joram et al. (1998) argued that students develop estimation ability and improve the strategies they use for estimation as they gain knowledge of physical measurement in higher grade levels, because knowledge and experience in real measurement is an indispensable requirement for measurement estimation.

Conversely, other studies (Swan & Jones, 1980; Montague & Van Garderen, 2003) have determined that students' performance in measurement estimation is not necessarily positively associated with a high grade level. For example, Swan and Jones (1980) observed that junior high-school students performed more favourably than high-school students in estimating long distances and metrically estimating the heights.

Moreover, Montague and Van Garderen (2003) compared the estimating ability of students who exhibited different mathematics abilities and levels of grade placement (Grades 4, 6, and 8). The results indicated that the fourth-grade children who received instruction based on a mathematics curriculum that reflected NCTM standards (NCTM, 2000) and that focused on measurement estimation outperformed the children in higher grades who used a different mathematics curriculum. The results of the study suggested that mathematics curricula and instruction may influence the estimation ability of children.

Children's Strategies for Performing Measurement Estimations

Forrester et al.'s (1990) and Chan's (2001) studies have determined that children frequently provide estimates by observing (visualizing) or guessing. Although visualization serves as the foundation for estimating, visualization is unlikely to generate a reasonable estimate without being facilitated by knowledge of measurement units and reference quantities. Furthermore, guessing may provide a gross estimate, but using this strategy without carefully recognizing the levels of reasonableness may yield poor estimates, such as substantial underestimates or overestimates.

Another approach, involving the use of benchmarks in which nonstandard units or events are used as referents for estimating may yield more accurate estimates than guessing does (Carter, 1986). Moreover, benchmarks that are constructed based on objects that are familiar to estimators can be more meaningful than standard units.

When children use standard units or benchmarks for estimating, they first decompose the objects to be estimated into samples for which basic estimate skills can be used (e.g., comparing and decomposing) and then recompose the objects to determine the estimated quantities (e.g., computing) (Carter, 1986; Forrester et al., 1990).

METHODLOGOGY

Participants

The sample consisted of 72 children (40 boys and 32 girls), each in one fourth- (n = 21), fifth- (n = 32), or sixth-grade (n = 19) class at local public elementary schools in cities in north Taiwan. The mean ages of the children in each grade were 10.19 years for Grade 4 (M = 122.24, SD = 3.63), 11.26 years for Grade 5 (M = 135.13, SD = 4.85) and 12.22 years for Grade 6 (M = 146.58, SD = 3.61). All of the participants had received instruction on length and area measurements, which was given based on the mathematics textbooks that reflected the guidelines for mathematics curriculum (TME, 2010) and that focused on measurement estimation, before participating in the study.

Instrument

In this study, an estimation task consisting of 12 problems that required estimating measures of length and area was designed by referring to textbook materials and the estimation tasks of Chan (2001). For example, "Estimate (without using a ruler) the length of the rope. (①4.2~5.0 m; ②6.1~7.0 m; ③5.0~6.0 m)" and "Estimate (without using a ruler) the length of the body of the caterpillar? ( ) cm." The objects of which the length or area was to-be-estimated in the problems were visually presented to the participants by using real objects or figures of the objects. The problems were divided into two subsets of six problems (i.e., subscales of linear estimation and area estimation). The estimation task was completed in 40 minutes.

The strategies that the children used for estimating linear measurement were collected from the participants' written answers to and interviews on an estimation problem that required estimating the length of a long object (5.6 meters). The interview, during which the participants were asked "What methods do you use for estimate the long rope?" was conducted after the participants completed the estimation task.

Scoring and category of estimation strategy

Numerical estimates were scored for accuracy and acceptableness (Siegel et al., 1982). "Accuracy" was defined as an estimate that was between plus 10% and minus 10% of the actual value. "Acceptableness" was defined as an estimate that was between plus 25% and minus 25% of the actual value. An "accurate" estimate was scored 2-points, whereas an "acceptable estimate" was scored 1-point. If an estimate was greater than plus 25% of the actual value or lower than minus 25% of the actual value, a score of "0" was allocated. The total score of each subscale was 12 points.

In this study, four types of estimation strategies that children tend to use for performing measurement estimation by referring to Forrester et al. (1990) and Chan (2001) were

used to classify the participants' strategies. The four types of strategies included: (a) Looking: "Looking" involves estimating the sizes of objects by using the naked eye (perception) without computation and using standard (or nonstandard) units to decompose and recompose the objects to be measured. (b) Guessing: A guessing estimate represents a gross estimate (Carter, 1986). An estimate that is generates by guessing without thinking properly about the correct answer needs to be recognized (and refined). Thus, a guessing estimate involves a conjecture. (c) Benchmark: Children select objects that are readily available in a classroom or body parts as references for estimation. (d) Other: This category contained a response of "Do not know" or implicitly describing a strategy or no answers.

The written answers for the estimation task of 26 children were independently scored by two raters. Regarding the reliability of the estimation problems scores, Pearson correlations indicated that the inter-rater agreement was $r = .98$, $p < .01$. Moreover, Kappa analyses were administered to test the reliability of the coding of the children's estimation strategy. The coding of the children's estimation strategy was assessed at .98, $p < .01$.

RESULTS

Table 1 presents the means and standard deviations of the performance of the children in the two subscales of measurement estimation and the entire estimation task according to grade level.

Types of measurement estimation	Grade 4			Grade 5			Grade 6			Total		
	n	*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*
Linear estimation	21	5.91	2.05	32	6.19	2.66	19	7.47	2.22	72	6.44	2.43
Area estimation	21	4.14	1.20	32	5.69	1.94	19	6.11	2.11	72	5.35	1.95
Entire estimation task	21	10.05	2.06	32	11.88	3.30	19	13.58	3.01			

Table 1: Means and Standard Deviations for the Children's Estimation Performance by Grade Level and Type of Measurement Estimation

To compare children's performance in the estimation task among grade levels, a one-way ANOVA was conducted. The results indicated significant differences in the performance of the children in the estimation task among the three grade levels, $F(2, 69) = 7.36$, $p < .01$, $\eta^2 = .18$. Schéffe post-hoc tests, used to analyze the differences in grade levels, indicated that the six-grade group significantly outperformed the fourth-grade group. The differences between the fifth-grade and fourth-grade groups were statistically nonsignificant. Furthermore, no differences were observed between the fifth-grade and six-grade groups.

Additionally, to compare performance yielded by the three grade levels in two types of measurement estimation were conducted by using 3 (grade level: Grade 4, 5, and 6) x 2 (estimation: linear and area estimations) within-subject analyses of variance (ANOVAs). Significant interaction effects were not found for the estimation

performance, $F(2, 69) = 1.18$. $p = .32$. The main effect of type of measurement estimation reached statistical significance, $F(1, 69) = 10.67$, $p < .01$, $\eta^2 = .13$. Moreover, the main effect of grade level also reached statistical significance, $F(2, 69) = 7.36$, $p < .01$, $\eta^2 = .18$.

Regarding the subscale of linear estimation, the results indicated that no significant differences among grade levels, $F(2, 69) = 2.50$, $p = .09$. Regarding the subscale of area estimation, the results indicated significant differences in the performance of the children among the three grade levels, $F(2, 69) = 6.91$, $p < .01$, $\eta^2 = .17$. Schéffe post-hoc tests, used to analyze the differences in grade levels, indicated that both the fifth- and sixth-grade groups significantly outperformed the fourth-grade group. However, no differences were observed between the fifth- and sixth-grade groups. This result was consistent with the results regarding the comparison of the entire estimation task.

The follow-up comparison on the differences between the scores of linear estimation and area estimation in each grade, the results indicated that the fourth-grade group obtained higher scores in the subscale of linear estimation than those in the subscale of area estimation, $F(1, 69) = 6.94$, $p < .01$. Such differences in the scores between the two subscales were not exhibited in the fifth-grade group, $F(1, 69) = .85$, $p = .36$, nor in the sixth-grade group, $F(1, 69) = 3.79$, $p = .06$.

Regarding the analysis of the strategies used by the children, because the children reported using multiple types of strategy, each type of strategy reported was coded and the total frequency of each category was calculated. The strategies consisted of four types: "looking," "guessing,""benchmark," and "other." The frequencies at which each strategy was used are ranked from high to low as follows: "Benchmark" (50 times), "Guessing" (9 times), "Looking" (8 times), and "Other" (5 times). For the use of "Benchmark," the children tended to use body parts (e.g., the length of fingers, the length between the index finger and thumb stretched, the feet, the palm of the hand, and the length of outstretched arms) and objects in the classroom (e.g., an eraser, a pencil, and the length of a tile on the ground of the classroom, and a blackboard). Regarding the "other" strategy, five children were categorized in the category, including two children who omitted to answer and three children who did not explicitly describe the approach they used (e.g., "measuring" or "drawing)."

DISCUSSION AND IMPLICATIONS FOR MATHEMATICS EDUCATION

This study examined children's competence in measurement estimations. The results of this study are summarized and discussed below. First, grade levels were related to differences in the measurement estimation ability of the children. For the entire estimation task, the six-grade children were more successful in performing measurement estimation than the fourth-grade children. The results of this study partially supported those of Siegel, Goldsmith, and Madson (1982) that reported that the children in Grade 6 provided more accurate estimates than did the children in Grades 2 to 5. The partial results that were inconsistent with those of Siegel's study

may result from the differences in curricula and instruction of school mathematics and the problem contexts (e.g., units and quantity) (Forrester et al., 1990).

Second, the factor of grade level also influenced the ability of children to estimate area. The children who were in fifth and sixth grades were more competent in estimating area than those in fourth-grade group. Moreover, both the fifth- and sixth-grade groups exhibited similar abilities for estimating area. The results suggested that, for the subscale of area estimation, the children in a higher grade level were more competent in area estimation than the children in fourth grade. However, the factor of grade level did not significantly influence the ability of children to estimate length. This finding is consistent with that of Forrester et al. (1990).

Overall, regarding the results on area and linear estimation, the fourth-grade children, who received instruction in linear and area measurement, could perform similarly to the fifth- and sixth-grade children in linear estimation but not in area estimation. The results may be caused by the differences in complexity between linear estimation and area estimation and the amount of experience in performing the two types of estimations. The process of area estimation is more complex than that of linear estimation (Chan, 2001; Sarama & Clements, 2009). Children require more knowledge of area measurement and experience in real measurement to make area estimation. Remarkably, the fifth-grade children performed equally well as the sixth-grade children did in the entire estimation task and the two subscales. However, the fifth-grade group did not outperform the fourth-grade group. This is probably because of the dissimilar approach of instruction on measurement that the participants received. Additionally, this implies that Grade 4 to 5 is a crucial stage at which the ability of children to perform measurement estimation, particularly, linear and area estimation, is developed. However, this assumption requires further investigations.

Finally, most of the children reported using one (or more) strategies for estimating an object with a long length; however, some children reported using of “looking” and “guessing” and “other” strategies. Compared with the results of Forrester et al. (1990) and Chan (2001) regarding the estimation strategies used by children, the results of this study indicated that the participants were inclined to use benchmarks for making estimation and were less likely to express “Do not know,” “Guessing,” or “thought.” These differences in strategy use may result from their experience in and knowledge of measurement that was obtained from school mathematics and everyday life.

Skill in measurement estimation (e.g., the use of strategies) can be improved through instruction (Joram et al., 1998). Grade level, which represents the amount of measurement experience acquired from school mathematics, may affect on the ability of children to measurement estimation. The more opportunities teachers provide for students to develop knowledge of measurement and experience in estimating, the more developed students’ abilities to measurement estimation may become.

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